

H.S.

Problem Sheet - 6

①
TRIGONOMETRY
MULTIPLE ANGLES

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1. Express $\cos 4\theta$ in terms of $\cos \theta$. [Ans: $8\cos^4\theta - 8\cos^2\theta + 1$]
2. Prove that $\sin^6 x + \cos^6 x = 1 - \frac{3}{4} \sin^2 2x$.
3. If $\sin A = \frac{3}{5}$ find the values of $\sin 2A$ & $\cos 2A$.
(i) If $\cos A = \frac{12}{13}$ find the value of $\sin 3A$.
(ii) If $\sin 2A = \frac{4}{5}$ find the value of $\tan A$ $[0 \leq A \leq \pi/4]$
[Ans: $\sin 2A = \frac{24}{25}, \cos 2A = \frac{7}{25}$]
[Ans: $-2035/2199$]
[Ans: $\frac{1}{2}$]
4. Prove that $2\cosec 4\theta - \sec 2\theta = \frac{1 - \tan \theta}{1 + \tan \theta} \cosec 2\theta$.
5. Express $u = \sin^6 x + \cos^6 x$ in the form $A + B \cos 4x$, where A & B are constants. Find A & B . Hence, obtain the max. and min. values of u . [Ans: $A = \frac{\sum B}{8} = \frac{3}{8}$; max value = 1; min value = $\frac{1}{4}$]
6. If $\tan x = \frac{b}{a}$ find the value of $(a \cos 2x + b \sin 2x)$.
7. Prove that $\cot \theta \cot 2\theta + \cot 2\theta \cot 3\theta + 2 = \cot \theta (\cot \theta - \cot 3\theta)$
8. If $2\cos \theta = x + \frac{1}{x}$ prove that, $2\cos 3\theta = x^3 + \frac{1}{x^3}$
9. If α & β are two acute angles and $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$
Show that $\tan \alpha = \sqrt{2} \tan \beta$.
10. Find the value of k from the following relations:
 $3(\cos 2\phi - \cos 2\theta) = 1 - \cos 2\theta \cos 2\phi$; $\tan \theta = k \tan \phi$
where θ & ϕ are acute angles.
11. Express $\cos 4\theta$ in terms of $\sin \theta$ [Ans: $1 - 8\sin^2 \theta + 8\sin^4 \theta$]
12. Prove that: $\cot 15^\circ - \tan 15^\circ = 2 \tan 60^\circ$
13. Prove that $\cos^2(45^\circ - \theta) - \sin^2(45^\circ - \theta) = \sin 2\theta$
14. Prove that $\tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$
15. Show that $\sin 8\theta = 8 \sin \theta \cos \theta \cos 2\theta \cos 4\theta$.
16. Prove that $\cos^6 A + \sin^6 A = 1 - 3\sin^2 A \cos^2 A = \frac{1}{4} (1 + 3\cos^2 2A)$

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17. Show that $\cos\theta - \tan\theta = 2\cos^2\theta$.

18. Show that $\cos^2(A-120^\circ) + \cos^2 A + \cos^2(A+120^\circ) = \frac{3}{2}$.

19. Prove that $\frac{\cos\theta - \tan\theta}{1 - 2\sin^2\theta} = \sec\theta \cosec\theta$.

20. If $\tan x, \tan y, \tan z$ are in GP, show that $\cos 2y = \frac{\cos(x+z)}{\cos(x-z)}$.

21. If $\tan\beta = \frac{\sin\alpha \cos\alpha}{2 + \cos^2\alpha}$, then prove that $3\tan(\alpha-\beta) = 2\tan\alpha$.

22. If $2\tan A = 3\tan B$ prove that $\tan(A-B) = \frac{\sin 2B}{5 - \cos 2B}$.

23. If $\tan(\alpha-\beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$ find the value of $\tan\alpha, \tan\beta$.

24. (i) $\cos\frac{\pi}{7} \cos\frac{2\pi}{7} \cos\frac{4\pi}{7} = -\frac{1}{8}$ (ii) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$

25. If $\cos^2\alpha - \sin^2\alpha = \tan^2\beta$, show that $\cos^2\beta - \sin^2\beta = \tan^2\alpha$.

26. $\tan^2\alpha = 1 + 2\tan^2\beta$, prove that $\cos 2\beta = 1 + 2\cos 2\alpha$.

27. Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$

28. Show that $\cos^2\theta + \cos^2(\alpha+\theta) - 2\cos\alpha \cos\theta \cos(\alpha+\theta)$ is independent of θ .

29. Prove that $\frac{\tan 2^n\theta}{\tan\theta} = \frac{(1+\sec 2\theta)(1+\sec^2 2\theta)(1+\sec^2 3\theta)\dots}{(1+\sec^2 n\theta)}$

30. (a) If $\theta = \frac{\pi}{2^n+1}$, then prove that:-

$$2^n \cos\theta \cos 2\theta \cos 2^2\theta \cos 2^3\theta \dots \cos 2^{n-1}\theta = 1.$$

(b) If $\theta = \frac{\pi}{2^n-1}$, then prove that

$$2^n \cos\theta \cos 2\theta \cos 2^2\theta \cos 2^3\theta \dots \cos 2^{n-1}\theta = -1.$$

31. Find the value of $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$.